Sheet 10 solution (Spaces and ADT)

1. The entities included in the vector (linear) space are scalars, points, and vectors.
   Some axioms that define how these entities are related:
   - Any two scalar can combined through addition or multiplication to produce another scalar
   - A point specifies a location and has neither a size nor a shape
   - There is no point-vector relations/operations
   - There is no point-point relations/operations
   - Two vectors can be added/subtracted to produce another vector
   - There are two types of vector multiplication: Cross product and Dot product

2. Euclidian space is different from vector (linear) space in that it adds the concept of a measurable distance between any two locations (points) in the space. Then it adds the axioms that two points are separated by a certain distance that can be measured or calculated according to the representation.

3. Affine space is different from Euclidian and vector (linear) spaces as follows:
   - It's and extension of Euclidean and vector (linear) spaces so it contains the same entities and define the same axioms that Euclidian space defines
   - It defines the point-vector addition: A point can be added to a vector to specify another point and consequently a vector can be specified by the subtraction of two points or in general two points can be added to produce another point under the conditions that the sum of their coefficients (scalars) is one.

4. In the context of scalars, point, and vectors spaces, an abstract space is a space that defines the sets of abstract entities in that space and defines the axioms that relate them. The entities are considered without tying it to any representation. The axioms are defined for the abstract entities, and then they must hold for any representation? In other words, the space define that facts and relationships between the entities regardless any representation used in specifying the entities or the details how these relations/manipulation are implemented in a given representation.
   While mathematicians think abstractly in terms of spaces, computer scientist thinks abstractly in terms of Abstract Data Types (ADT). An ADT is the specification of the facts and the allowed operation/manipulation about a given type regardless how the instances of the type are represented in memory or how these operation and manipulation are performed for a given representation.

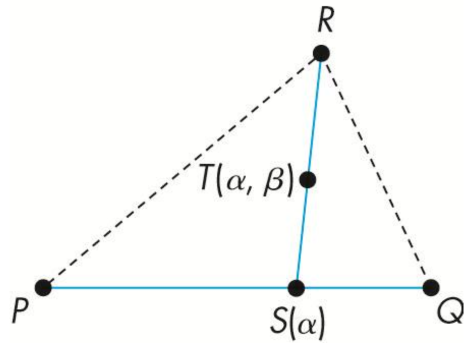
5. Vectors are directed quantities. They are used to represent real world directed quantities such as force, displacements and velocities. Geometrically, we have no such quantities but vectors are still applicable to specify **directed** line segments in space.

6. Convexity of a geometric object: A convex object is one for which any point lying on the line segment connecting any two points in the object is also in the object. Convexity is an important property in computer graphics

The convex hull of a set of points $P_1, P_2, \ldots P_n$, is the set of point defined using he following equation:

$$P = \propto_1 P_1 + \propto_1 P_1 + \cdots + \propto_1 P_1, \quad \propto_1 + \propto_1 + \cdots + \propto_1 = 1, and \ \propto_1 \geq 0$$

7. Consider the following figure and using the parametric for of a line segment we have



$$S(\propto) = \propto P + (1 - \propto)Q, \quad 0 \leq \propto \leq 1$$
$$T(\beta) = \beta S + (1 - \beta)R, \quad 0 \leq \beta \leq 1$$
$$T(\propto, \beta) = \beta[\propto P + (1 - \propto)Q] + (1 - \beta)R$$

The points defined the two parameters are points on the unique plane that includes the given three points P, Q, and Q.

8. Real-world objects are three dimensional objects but they have three characteristics that fit them with existing graphics hardware that deals with simple planner geometric objects
   - The objects are described by their surfaces and can be though as being hollow
   - The objects can be specified through a set of vertices in 3D
   - The objects either are composed of or an be approximated by flat, convex polygons